Control of a Cantilever Array by a Periodic Network of Resistances

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Abstract In this paper, we present a two-scale model including an optimal active control for a one-dimensional cantilever array with regularly spaced actuators and sensors. With the purpose of implementing the control in real time, we propose an approximation that may be realized by an analog distributed electronic circuit. More precisely, our analog processor is made by Periodic Network of Resistances (PNR). The control approximation method is based on two general concepts, namely on functions of operators and on the Dunford-Schwartz representation formula. We conducted validations of the control approximation method as well as of its effect in the complete control loop.

Two-Scale Model of Cantilever Arrays

Assumptions
- Thin elastic domain
- Periodic structure
- Cantilever thickness << Support thickness

Applying the two-scale approximation method, we get the two-scale model governing deflection for an infinite number of cantilevers:

\[ \rho^C \partial_{tt}^C w + R^C \partial_t^C w = u, \]
\[ \rho B \partial_{tt}^B w + \rho \omega^2 \partial_t^B w = \alpha \]

In base
\[ \rho^B \partial_{tt}^B w + B^D \partial_t^B w + \omega^2 \partial_t^B w = u \]
At their interface \[ \partial_y w = 0 \]

Semi-decentralized Control

The model is discretized in the cantilever direction, so only the PDE remains in the base direction. Then, it is written under in the form of a state space formulation, and the optimal control problem for vibration damping is stated as a LQR problem. Its solution uses the solution P of a Riccati equation:

\[ \frac{dz}{dt}(t) = Az(t) + Bu(t) \quad \text{for} \quad t > 0, \quad z(0) = z_0, \]
\[ J(z_0, u) = \int_0^{\infty} \| Cz \|_2^2 + \langle Ru, u \rangle \, dt, \quad \min_{u \in U} J(z_0, u), \]
\[ A^* P + PA - B R^{-1} B^* P + C^* C = 0 \quad u^* = -K z, \quad K = S^{-1} B^* P \]

The Riccati operator P is not local:

Function of an Operator or Matrix

\[ \partial_t^2 w = f \quad \text{in the base and} \quad w = \partial_y w = 0 \quad \text{at its boundary} \]

\[ \Lambda : f \rightarrow w \]

Definition of a function k of \( \Lambda \)

\[ k(\lambda) = \sum_{\ell=1}^{M} c_\ell \lambda^\ell \]

Is solution to the system

\[ \zeta_1 c_{1} v_1^\star - \zeta_2 c_{2} v_2^\star - \Lambda v_1^\star = 2 \pi \sum_{\ell=1}^{M} c_\ell \lambda^\ell \]
\[ \zeta_2 c_{1} v_1^\star + \zeta_1 c_{2} v_2^\star - \Lambda v_2^\star = 2 \pi \sum_{\ell=1}^{M} c_\ell \lambda^\ell \]

Function of an Operator by a PNR

After space discretization by a Finite Difference Scheme

Control by a Function of an Oper.

We perform changes of variable with « simple operators » \( \Phi_Z, \Phi_U \) to find that the control operator is expressed as a composition of \( \Phi_Z, \Phi_U \) and a function of an operator:

\[ K = \Phi_U \Phi_Z (\lambda) \Phi_Z^T (\lambda) \]

Numerical Validations

Errors on P with respect to M the number of quadrature points and L the number of nodes in the finite difference scheme:

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<th>30</th>
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</table>

From our first investigations, applying the approximated control is effective in the loop with the two-scale model.

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