Abstract. In [1] and [2] we developed a theoretical framework of diffusive realization (DR) for state-realizations of some linear operators. Those are solutions to certain operator linear differential equations in one-dimensional bounded domains. We also illustrated the theory and developed a numerical method for a Lyapunov equation arising from optimal control theory of the heat equation. However, the principles of our numerical methods were only sketched, and now we provide more details. Then, we do not only provide validation results of the method, but we also report our experience in its implementation on a Field Programmable Gate Arrays (FPGA), for the purpose of promoting embedded real-time computation.

**Numerical results**

Lyapunov equation: $\int_0^1 d\xi (\frac{d^2}{dx^2} y(x,\xi)) + \frac{d}{dx} (\frac{d}{dx} y(x,\xi)) dx = \int_0^1 Qu \; dx$, for all $u, v \in H^1(\Omega)$

**DR approximation:**

- Discretization of $y(x,\xi)$:
  - $h = 0.005 \ldots 0.1$
  - Constant approximation of $u = L$
- Choice of contour:
  - Parabola contour $= P$
  - Hyperbola contour $= H$
- Calculation of $\mu^{N}$: $N = (15,15)$

**Computation time:**

- $N = (15,15)$
- Constant approximation of $u$
- Parabola contour $= P$
- Hyperbola contour $= H$
- $D$ = numerical integration of $\int p(x,y) u(y) \; dy$ by trapezoidal rule

**Algorithm, FPGA implementation**

**Algorithm:**

1. Offline Computation of diffusive symbol $\mu^{+}(x,\xi)$
2. Online Computation
3. for $n = 0, \ldots, N$ do
4. for $k = 1, \ldots, M$ do
5. $\psi_{n+1}^k = \psi_{n+1,k} + \psi_{n,k} u_n$,
6. end for
7. $\psi_{n+1} = 2h \Re \left\{ \sum_{k=1}^M \psi_{n+1,k} \left( a_k v_{n,k} + b_k u_n \right) \right\}$
8. end for

**FPGA implementation:**

- For $M = 4$ and $N = 8$
- FPGA: 100 MHz, PC: 1.6 GHz.
- PC: $\approx 60000 \text{ ns}$
- FPGA:
  - parallel: 720 ns
  - pipeline: 359 ns
  - sequential: 1280 ns
- Ratio: 83 to 167 fold improvement,
- Bandwidth: 100 Mb/s.

**Computation**

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**Conclusion**

We have shown that the DR is well suited for FPGA implementation of the realization of a linear operator which plays a central role in a distributed control law. We have developed every steps to analyze computation accuracy. In particular, we have tested data formats consistent with hardware implementation. Our results proved a high efficiency in terms of computation time. However, using a Spartan3A FPGA, the amount of available values for constants and for inputs $u$ is limited by the number of logical cells.

**References**


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